**ME 8710: Final Exam**

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**Introduction to Problem**

Under the given problem for the mentioned constraints, an optimal solution is inbound for low-Earth orbit Applications' desired operating conditions (Altitude < 2000Km). The problem is a typical orbital rocket configuration plan for the given altitude.

The configuration designed for the given application and constraints needs to be tested onto the simulation developed in C++. The methodology adopted to lead a solution for an optimal condition has been defined by the various headers file equations.

In the direction to lead an optimal solution, different approaches have been utilized. Some of the strategies that failed while finding the solution have also been mentioned in the report's further sections, with the potential cause of their failure. The next section of the report highlights the operational constraints given for the above configuration problem.

**Problem Definition**

The problem has defined four operational constraints to estimate the viability and optimality of the rocket launch system. Wherein the operating conditions of the following system are:

**A. Payload Capacity Constraint:**

The design should be such that it can carry a payload of at least 1000kg and less than 15000kg. The payload mass is a significant constraint when defining a given space program's overall efficiency and objectivity.

In usual space programs, the payload could be a satellite, space probe, or spacecraft carrying cargo, humans, etc. The payload is the total mass exclusive of the wet mass and the rocket's dry mass component.

**Wet Mass:**

The rocket's wet mass can be defined as the total initial mass of the rocket that includes the mass of the propellent.

**Dry Mass:**

The rocket's dry mass can be defined as the mass of the rocket at its full ascent.

The payload needs to be defined for the given range, and the rocket configuration should be able to place the payloads into an orbit at least 400 km in altitude.

**B. Dynamic Pressure Constraint:**

The maximum dynamic pressure in which the entire configuration should operate has to be less than 33% of the Atmospheric Pressure, i.e., 33,400 Pa. The maximum dynamic pressure condition defines the point of Max Q.

Max Q is a phenomenon wherein at a point where dynamic pressure change due to increasing the velocity is more significant than due to decreasing air density i.e.

Change in Dynamic Pressure (Increasing Velocity) > Change in Dynamic Pressure (Decreasing Air Density)

This Max Q phenomenon opposes the kinetic energy of the given system. On the contrary, the opposite holds when the system passes over the point of Max Q. Many configurations for this very reason are throttled down near the Max Q periphery to avoid opposition to the system's overall Kinetic Energy.

The ideal throttle-down process should hit the Max Q, and the following considerations have been duly considered to define an optimal configuration for this problem.

**C. Maximum Acceleration Constraint:**

The overall acceleration profile has been limited to 8g, where g is the gravitational acceleration (g =9.8054 m/s2 as defined in the *constants.h*). The g-force component is due to gravitational pull, the gravitational acceleration, i.e., g changes with altitude change and remains constant on the earth's surface.

Chart

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Figure 1: STES 121 acceleration profile

Typically, A STES 121 ascent profile observed in the above figure is experiencing a maximum of 30m/s2 of acceleration from the lift-off stage to the engine's cut-off stage.

**D. Orbital Acceleration Balance Constraint:**

The orbital acceleration balance is a very significant factor that determines the rocket's ability to enter a stable orbit.

As we know, the thrust generated is to overcome the weight of the rocket. The orbital acceleration is the difference between the centripetal acceleration and acceleration due to gravity.

Orbital Acceleration Balance = Centripetal Acceleration - Acceleration due to gravity (h)

where,

rOverall = Radius of Earth + Final Altitude at a given time instant + Initial Altitude

Acceleration due to gravity is a function of altitude/height(h). It decreases with an increase in altitude.

Ideally, the OAB should be equal to zero. The configuration has been specifically deduced to attain a value in positive domain to maintain and attain a stable orbit with minimum energy requirement.

In this problem, the current formulation does not account for the tangential acceleration component. It does not account for the pitch-over, and only it has the radial component of the acceleration. The orbital acceleration balance profile, if positive, indicates the rocket will attain and maintain a stable orbit.

**Variables in the Problem**

The variables in the given configuration problem, which are user-defined, are:

* Number of Stages
* Type & Number of Booster required (If any)
* Type & Number of First (Main) Stage Engines
* Type & Number of Second Stage Engines (Engines for the second stage are used for the successive stages after the main stage)
* Throttle Profile for Main Stage and Boosters
* Burn Time for different stages in the configuration
* Main Core Diameter
* Payload Carrying Capacity (1000 kg < Payload < 15000 kg)

**Problem Objective**

To work with maximum possible payload configuration and equipping highest efficient rocket engine profiles.

**Ideal Approach for Problem Formulation**

The ideal formulation of the problem would begin with deciding the desired altitude for the given program and the total time of flight, which will work as a constraint to choose from the array of options of choosing from rockets providing different thrust. The ideal formulation has been adopted from Pathways to Exploration as quoted in reference section [1].

In each program, selecting a rocket engine over another, highly depends on the propellent budget, as you have to account for the energy requirements to pull the additional propellant mass as per the program's desired flight time.

**Ideal rocket equation/Tsiolkovsky rocket equation:**

This relation analogy can be observed consistent with the ideal rocket equation, using a mass fraction (mo / mf).

As mentioned in the further section about Hohmann Transfers, they are used to deduce efficient path planning solution for a given source and destination.

**Diagram, schematic

Description automatically generatedHohmann** **Transfers**

Figure 2: Hohmann Transfer Representation

In Hohmann transfer orbit, an elliptical orbit is used to lead transfer between two orbits even with different radii around a standard central body frame. The Hohmann transfer's need emerges from the need to deduce a practical path for the flight duration. Hohmann transfer helps to use the lowest possible amount of propellent to travel between the source and the desired destination.

The above approach encompassing drag, air resistance, and minimum increase in elevation are some of the non-ideal factors, enabling to effectively lay the bounds of the problem and a fecund mathematical model to carry out the optimization for the desired constraints.

Though the above modeling approach is ideal, it does not work for the current scenario, as the problem formulation of this kind needs a lot of instantaneous & comprehensive data profiles for different constraints.

**Failed Approach**

The initial approach utilized the ideal rocket equation to work through the equations and formulate the problem as per the defined problem definition.

Where,

Now, It can be also be written in the form of

Where,

The above mass model can be further modeled for the change in mass as defined below, wherein the q(t) i.e the flow rate of expelled gas is equivalent to change in mass for the given system.

If rocket experiences gravity, the total force on the rocket is equivalent to –

Where, g = gravitational acceleration (m/s2)

The force on the exhaust is its flow rate multiplied by the velocity component

Let, w be the velocity of the exhaust relative to the rocket

In this case, F= ma idealization would deem wrong, as mass(m) changes with respect to time. Again, this approach was deemed wrong if mass taken constant due to fixed mass idealization. With insufficient data profiles could not deduce instantaneous mass calculations. Though, even above equations needed more comprehensive data profiles to lead the calculations further.

Even the further instantaneous mass calculations were not coherent with that of the provided simulation suite, so this approach was also dropped. Further, an ideal solution could begin with Euler Equation formulation of the above problem, even that led to inaccurate calculations. A system in place of the above, if had modelled the changed in momentum then the formulation might be correct for the instantaneous component rather than the

F = mass \* acceleration

Furthermore, the many researchers [2] recommends utilization of Heun’s Rule for more accurate approximation and calculations.

**Objective Formulation Pedagogy**

After an unsuccessful formulation, in this final formulation section, the problem has been formulated to deduce feasible configurations for the highest payload for the user-defined altitude. The user-defined altitude will set the upper bound over the altitude profile of the given problem.

* **Designed Payload Capacity = 15,000 kg (Agile & Equivalence Constraint)**
* **Desired Altitude >= 1000 km (Agile Constraint)**
* **Minimum Altitude Constraint = 400 km (Given Constraint)**

Primarily, the velocity criterion has been calculated to establish the minimum velocity required by the rocket engine profiles to satisfy the given problem's operational constraints and requirements.

Now, from the equations of motion, we can equate the orbital acceleration balance equation as mentioned earlier in the section to zero, we get –

7378 km

m = 6778 km

Equivalent equation for velocity can be written as

Case – 1 (With )

The vmin puts a constraint over the minimum velocity required to achieve the corresponding desired altitude.

After defining the velocity criterion, the burn time for the entire configuration duration has been calculated using the 8g constraint of acceleration.

* **Acceleration <= 8g (Given Constraint)**

Possible Range of Acceleration permitted as per the laid constraint:

g operating range considerations

* Burn Time Validation
* Objective Formulation Pedagogy
* Two different Objectives, first, objective function for the rocket selection based on three different objectives. Second type of objective function is for main core diameter selection.

- Objective – Rocket Selection (First Approach) - Selection Metrics\*\*

A. Selection Criteria I

B. Selection Criteria II

- Objective – Rocket Selection (Third Approach) -

(Paper)

**Objective – Main Core Diameter Selection**

**Results & Discussion**

**Conclusion**

**References**

1. Book: A National Research Council, Pathways to Exploration: Rationales and Approaches for a U.S. Program of Human Space Exploration, 978-0-309-30507-5 (2014)
2. *Euler Equations for Rockets*. (2014, June 12). NASA Web Portal. https://www.grc.nasa.gov/www/k-12/rocket/eulereqs.html